The Role of Risk Capital in Public-Private Partnerships

IMaR 20 Oct 2022

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Summary of talk

- The different roles of the public and private sector in PPPs
- Risk capital versus investment capital
- Risk capital and asset/payoff allocation
- The impact of risk capital on the effective cost of financing
- The impact of project volatility on risk capital
- Construction of exit strategies for the private sector

- We start with a brief discussion of PPP

Introduction

- PPP are large scale projects based on a co-operation between the public and the private sector
- Generally, the public sector plays the role of a facilitator, whereas the expertise and the know how are delivered by the private sector
- Projects undertaken are large scale, require very substantial capital investments and are risky but, at the same time, likely to provide substantial social benefits
- The high risk and the amount of capital required make some of the considered projects out of reach for private enterprise alone
- Similarly, the required level of knowledge and technical expertise make the projects non feasible to undertake by the public sector alone – public and private come together in a kind of symbiotic relationship

Introduction

- Projects are based on and structured in line with the respective strengths of the public and the private sector
- Generally:
  - Public sector – provider of risk capital and guarantees
  - Private sector – provider of investment capital, expertise and knowhow essential for success

- The basic concept of PPP is very simple – but – the details of can be very complex

Introduction

- Identification, quantification and the management of the risks associated with large scale research and development projects is generally difficult task – as many projects are unique
- However, if there are some measures in place that can put floor under suffered losses a private/public partnership may be easier to facilitate
- That is where the risk capital and the role of the public sector come in

Risk capital – basic definition

- Risk capital is defined as the price of risk – i.e. the cost of removing financial risk
- Risk capital can be viewed as the cost (premium) that makes it possible to finance a project with risk free debt – i.e. the cost of debt is the risk free rate
- An important role of the public sector in PPPs is to provide risk capital (RC) whereas the role of the private sector is to provide the investment capital (IC)
- The ratio $\eta = RC/IC$ is project dependent

Risk capital – model assumptions

- A project is financed by a loan of nominal value IC_0
- At time $t = 0$ the value of the project is equal to the required investment $V_0 = IC_0$
- At time T the value of the project is V_T; V_T is presented by a stochastic variable
- At time T loan payback is required – or ability to do so

Risk capital – model assumptions

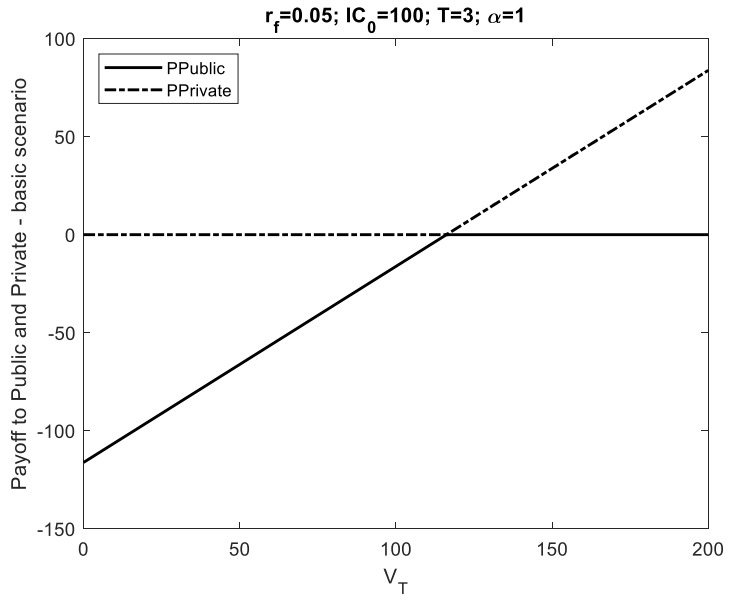
- The private enterprise borrows the investment capital IC_0 from some financial institution or fund
- The public sector provides the risk capital that enables the private enterprise to borrow at the risk free rate

Risk capital – basic definition

- The investment capital is provided by some financial institution or investment fund – possibly pension fund or a hedge fund?
- The public sector provides the risk capital
- Possible scenario - outcomes at future time T

Case	Lender	Public Sector	Private Sector	Total
$V_T > IC_T$	IC_T	0	$V_T - IC_T$	V_T
$V_T < IC_T$	IC_T	$-(IC_T - V_T)$	0	V_T

- Losses go to the public sector – gains go to the private sector!
- This provides a base-scenario which we built on

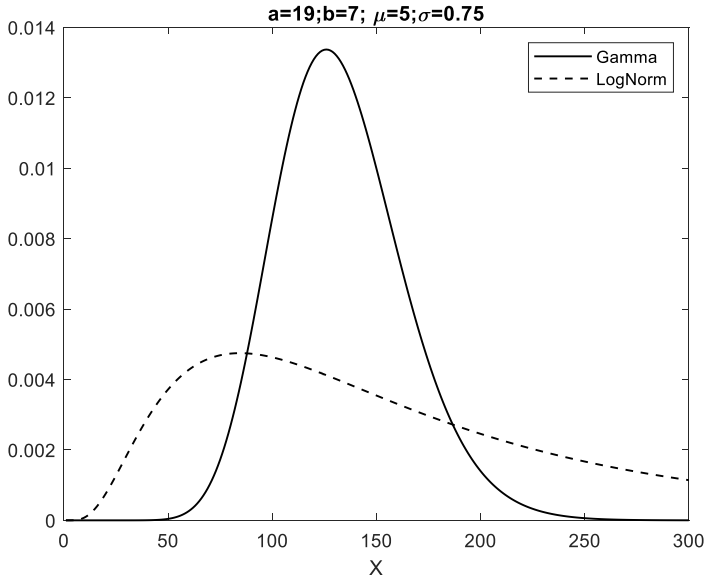


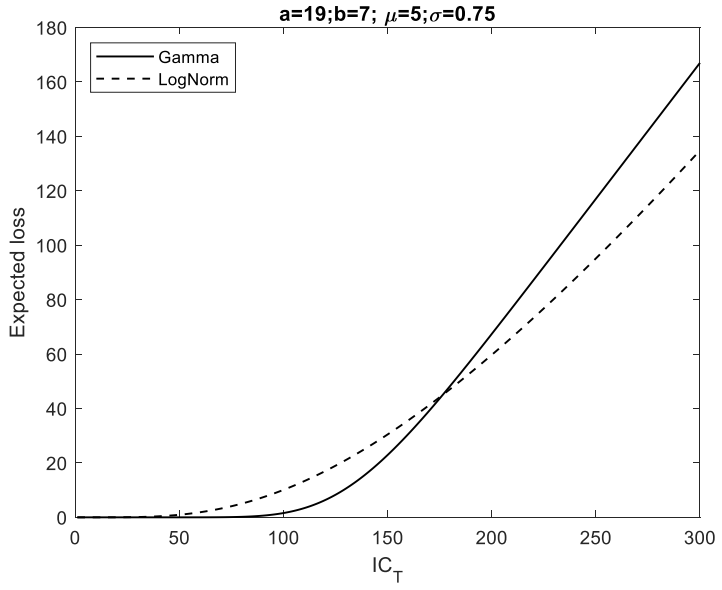
Risk capital - evaluation

- The expected public sector liability depends on the stochastic assumptions made
- The expected value of the risk capital is

$$EL_T = \mathbb{E}_t \max\left(IC_T - V_T, 0\right) = IC_T F\left(IC_T\right) - \mathbb{E}_t\left(V_T \middle| V_T \le IC_T\right)$$

- The evaluation of this expression depends on the observed or assumed distribution for V_T





Program: GammaIntegral

Different project value distributions

- Weibull distribution:

$$\mathbb{E}\left(\max\left(IC_T - V_T, 0\right)\right) = \alpha\kappa\left\{\int_0^{IC_T}\left(IC_T v^{\alpha-1}e^{-\kappa v^{\alpha}} - v^{\alpha}e^{-\kappa v^{\alpha}}\right)dv\right\}$$

- Pareto distribution:

$$\mathbb{E}\left(\max\left(IC_T - V_T, 0\right)\right) = IC_T F\left(IC_T\right) + \sigma_{V_T}^2 \frac{\left(\beta-1\right)^2\left(\beta-2\right)}{\beta+1}\left(IC_T^{-(\beta+1)} - \alpha^{-(\beta+1)}\right)$$

- Gamma distribution:

$$\mathbb{E}\left(\max\left(IC_T - V_T, 0\right)\right) = \frac{\lambda^{\beta}}{\Gamma\left(\beta\right)}\int_0^{IC_T}\left(IC_T - v\right)v^{\beta-1}e^{-\lambda v}dv$$

- Log normal

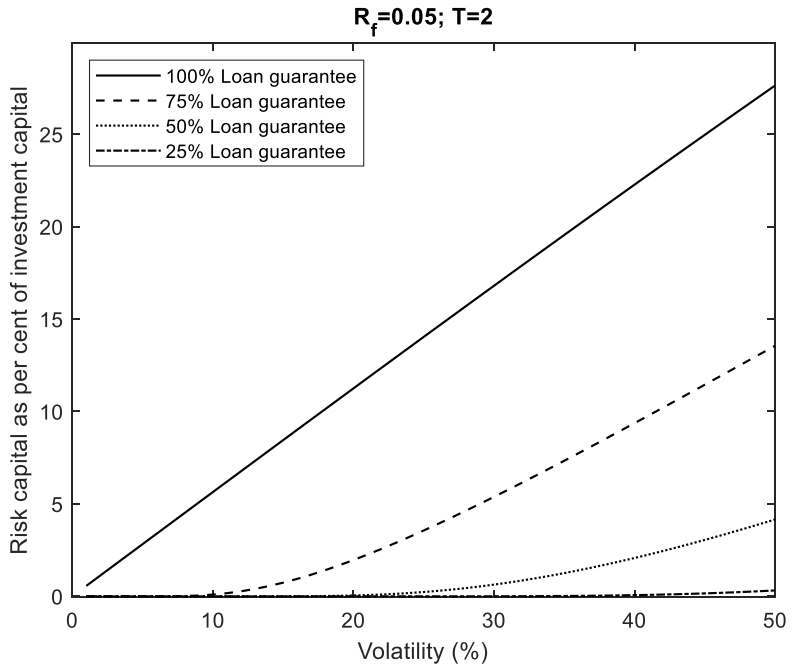
$$\mathbb{E}\left(\max\left(IC_T - V_T, 0\right)\right) = IC_T N\left(-d2\right) - e^{r_f T}V_t N\left(-d_1\right)$$

- The Weibull and the Gamma cases can only be solved numerically

Risk capital for limited protection

- Risk capital as a percentage of investment capital for different protection cover

$$
\begin{aligned}
EL_T &= \mathbb{E}_t \max\left(\alpha IC_T - V_T, 0\right) \\
&= \alpha IC_T F\left(\alpha IC_T\right) - \mathbb{E}_t\left(V_T \middle| V_T \le \alpha IC_T\right)
\end{aligned}
$$

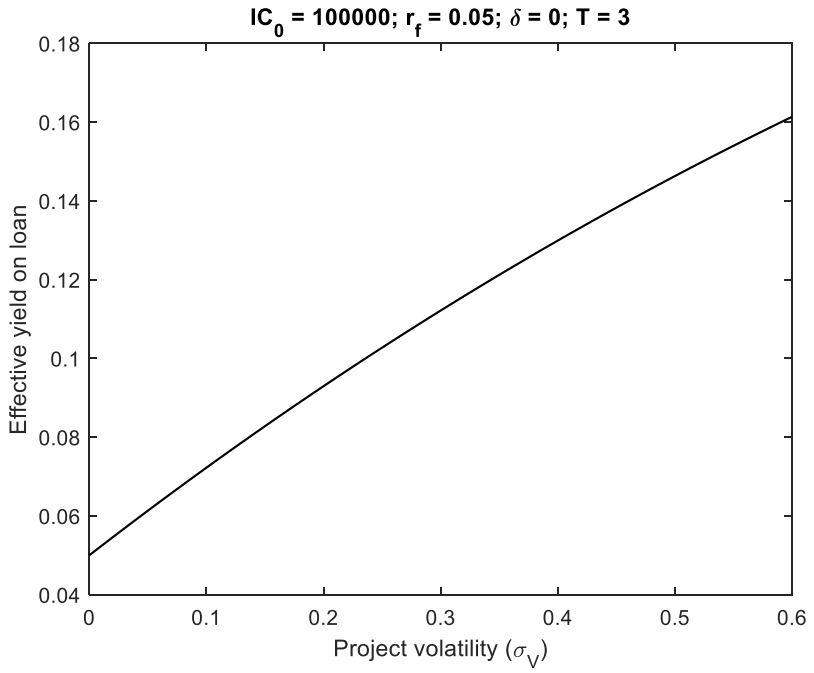


Program: FracRiskCap

Risk capital – the effective cost of financing

- The effective cost of financing r_c can be estimated from

$$IC_0 e^{r_c T} = IC_0 e^{r_f T} + PV\left(EL_T\right) e^{r_f T} \quad \Rightarrow \quad r_c = r_f + \frac{1}{T}\log\left(1 + \frac{PV\left(EL_T\right)}{IC_0}\right)$$



Program: CostOfRiskFreeLoan

Comments on risk and utility

- Our measure of risk exposure is the estimated or implied volatility of asset/project returns
- This risk is modelled by the probability distribution for V_T and impacts on the expected loss and therefore the amount of risk capital required
- Another measure, sometimes used to quantify risk, is a utility function $U(x)$
- Let w be present value of wealth, η zero mean risk and π the premium required to account for the risk. Then

$$\mathbb{E}\,U\left(W+\eta\right)=U\left(W-\pi\right)$$

- Taylor expanding both sides – using $\mathbb{E}\eta=0$ gives

$$u\left(w\right)+\frac{1}{2}\sigma_\eta^2 u''\left(w\right)=u\left(w\right)-\pi u'\left(w\right)\;\Rightarrow\;\pi\approx\frac{1}{2}\sigma_\eta^2 A\left(w\right)$$

where

$$A\left(w\right)=-\frac{u''\left(w\right)}{u'\left(w\right)}$$

is the so called absolute risk aversion coefficient

Comments on risk and utility

- The problem with the utility approach is that it does not provide an objective measure of risk
- Two parties can have different objective functions

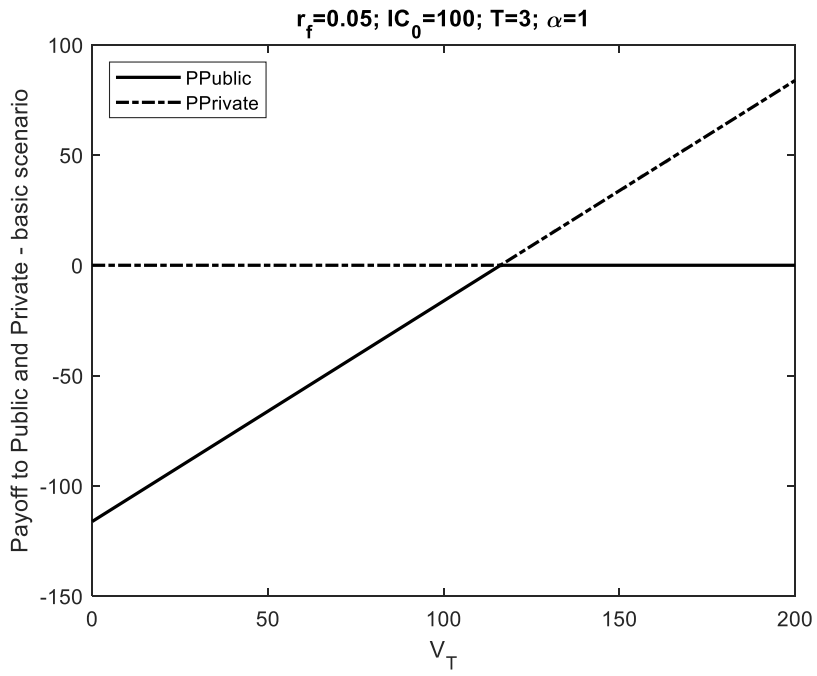
Asset allocation

- **Important question**: How should asset ownership/payoff be allocated between the public and the private sector
- Should the public sector be given an option on the project value for providing risk capital?
- How should the strike for such an option be decided?

Risk capital – who provides it?

- Basic scenario – the private sector receives all the payoff – the public sector picks up the losses!
- Generally – not an acceptable scenario

$$P_{pub} = -\max\left(IC_T - V_T, 0\right)$$
$$P_{priv} = \max\left(V_T - IC_T, 0\right)$$



[PPublic,PPrivate] = PayoffsInPPP(0:1:200,100,0.05,3,75,175,1,0.35,0);

Risk capital – who provides it?

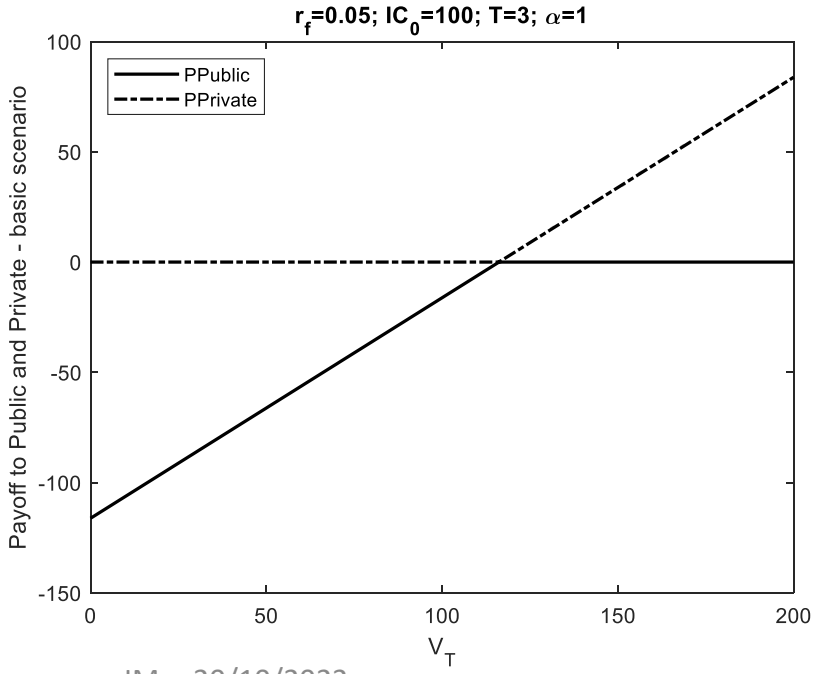
- On previous slide it has been assumed that all the risk capital comes from the public sector
- We argue that also some of the risk capital should come from the private sector

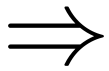
Fair loss/gain distribution

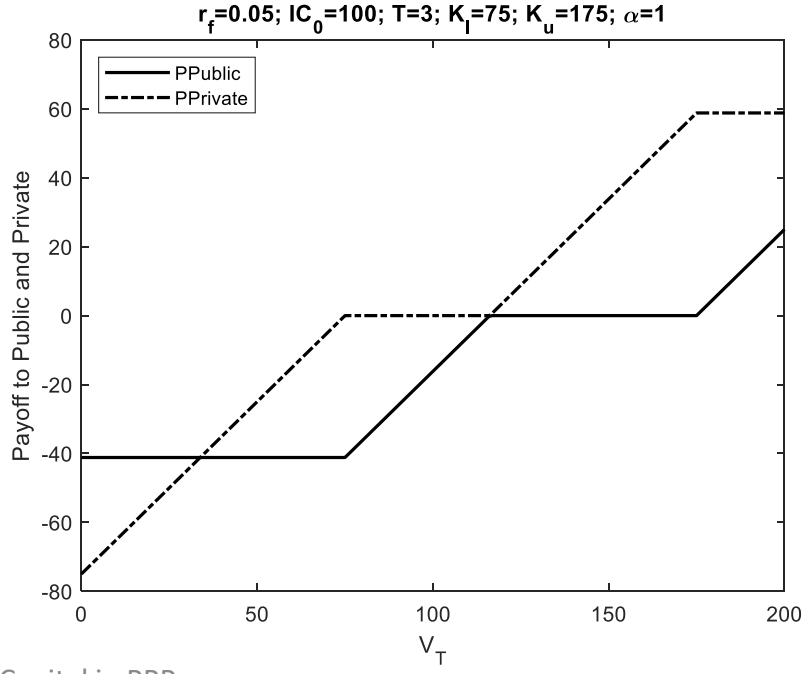
- In a modified version of the model the private sector picks up some of the losses (below $K_l = 75$) and also gives up some of the gains (above $K_u = 175$)

$$P_{pub} = -\max\left(IC_T - V_T, 0\right)$$
$$P_{priv} = \max\left(V_T - IC_T, 0\right)$$

$$P_{pub}\left(T\right) = -\max\left(IC_T - V_T, 0\right) + \max\left(V_T - K_u, 0\right) + \max\left(K_l - V_T, 0\right)$$
$$P_{priv}\left(T\right) = \max\left(V_T - IC_T, 0\right) - \max\left(V_T - K_u, 0\right) - \max\left(K_l - V_T, 0\right)$$







[PPublic,PPrivate] = PayoffsInPPP(0:1:200,100,0.05,3,75,175,1,0.35,0);

Fair pricing of financial instruments

- Let V_t be a random variable presenting the price of an asset
- Define the payoffs for two contracts, at the future time T as

$$\Psi_c\left(T\right) = \max\left(V_T - K_c, 0\right) \;;\; \Psi_p\left(T\right) = \max\left(K_p - V_T, 0\right)$$

- The value of these contracts today are

$$\Psi_c\left(0\right) = \mathbb{E}_t\left(D\left(0,T\right)\Psi_c\left(T\right)\right) \;;\; \Psi_p\left(t\right) = \mathbb{E}_t\left(D\left(0,T\right)\Psi_p\left(T\right)\right)$$

Fair loss/gain distribution

- The present value of public and private positions

$$P_{pub}\left(0;K_l,K_u\right) = -put\left(IC_0,\alpha IC_T,r_f,T,\sigma,\delta\right) + call\left(IC_0,K_u,r_f,T,\sigma,\delta\right) + put\left(IC_0,K_l,r_f,T,\sigma,\delta\right)$$

$$P_{priv}\left(0;K_l,K_u\right) = call\left(IC_0,\alpha IC_T,r_f,T,\sigma,\delta\right) - call\left(IC_0,K_u,r_f,T,\sigma,\delta\right) - put\left(IC_0,K_u,r_f,T,\sigma,\delta\right)$$

- A fair asset allocation seeks the pair $\mathbf{K}^* = (K_l^*, K_u^*)$, such that

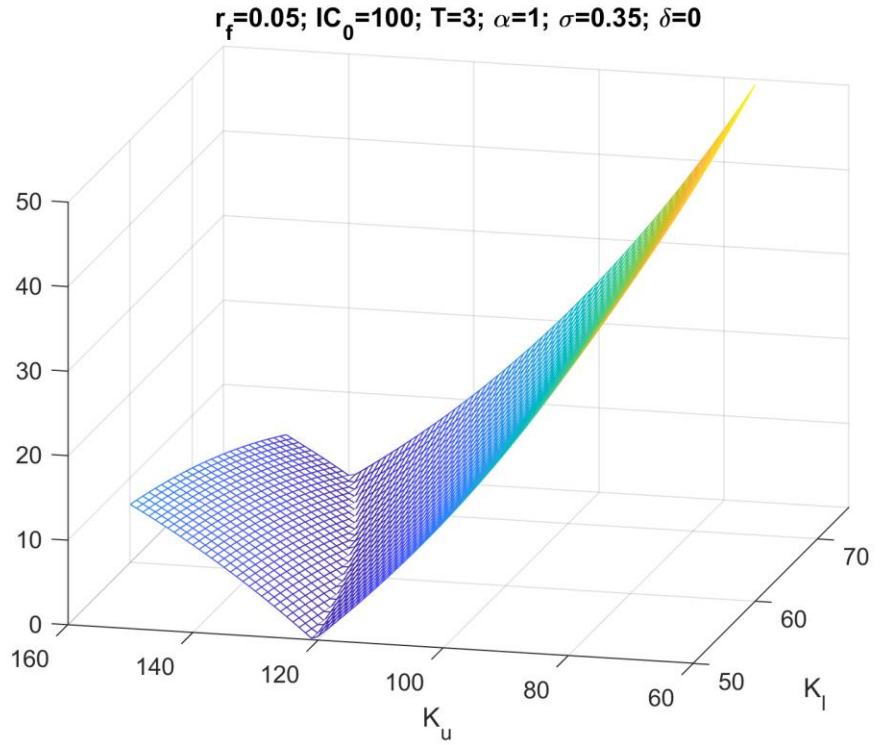
$$P_{pub}\left(0;K_l^*,K_u^*\right) = P_{priv}\left(0;K_l^*,K_u^*\right)$$

Fair loss/gain distribution

- We make K_l and K_u run independently over a range of values and consider the expression

$$DiffValue\left(K_l, K_u\right) = \left[\left(P_{pub}\left(K_l, K_u\right) - P_{priv}\left(K_l, K_u\right)\right)^2\right]^{1/2}$$

- We surface plot the result



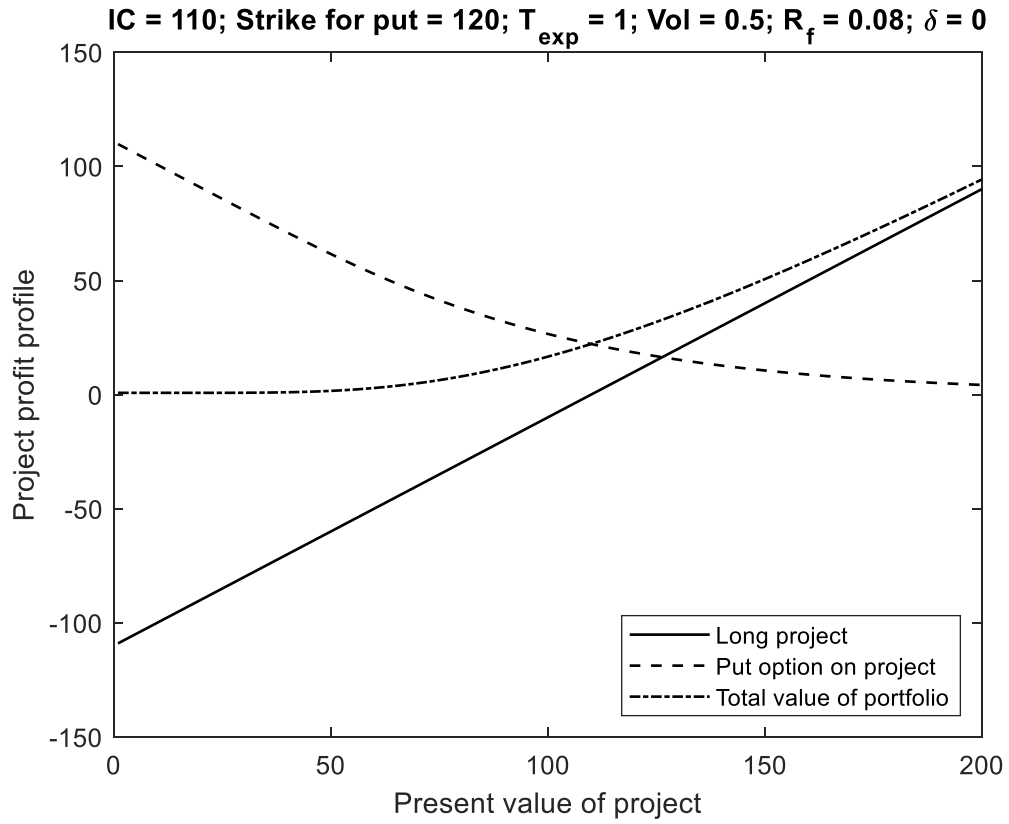
[PPublic,PPrivate,DiffValue] = PPPEqualitySurface(100,0.05,3,50:1:75,75:1:150,1,0.35,0);

Providing an exit strategy for the private sector

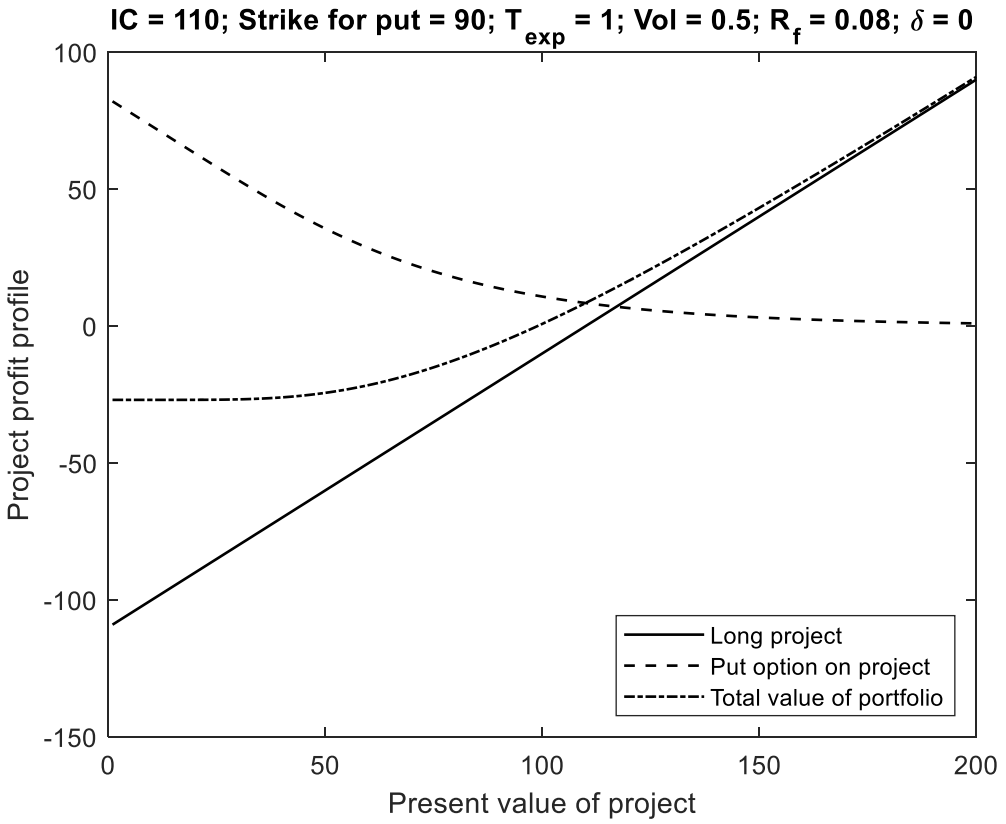
- The public sector can offer the private sector an exit strategy from a project by writing a put option on the present value of the project's cash flow
- Essentially the public sector commits to buying the residual value of the project for a prefixed strike value – in case the project is not successful
- The private sector's long position in the project is extended by a long position in a put option

Providing an exit strategy for the private sector

- The public sector writes a protective contract for the private sector – with different floor levels



[PVCF,NPV,Put] = ProjectOption(1:1:200,110,0.08,1,0.5,120,0.1);



[PVCF,NPV,Put] = ProjectOption(1:1:200,110,0.08,1,0.5,90,0.0);

Summary

- Provision of risk capital makes it possible for the private sector to borrow at a lower interest rate
- Risk capital depends strongly on estimated project risk and assumed distribution of project value
- Risk capital makes it possible to quantify the contribution of the public sector to PPPs
- We have devised a new way to split the risk capital between the public and the private sector
- Risk capital provides a basis for fixing a fair distribution of project payoffs between the public and the private sector
- We discussed different risk measures based on assumed utility or project volatility